4 - Le(s) grand(s) au-delà

4.1 - La grande unification?

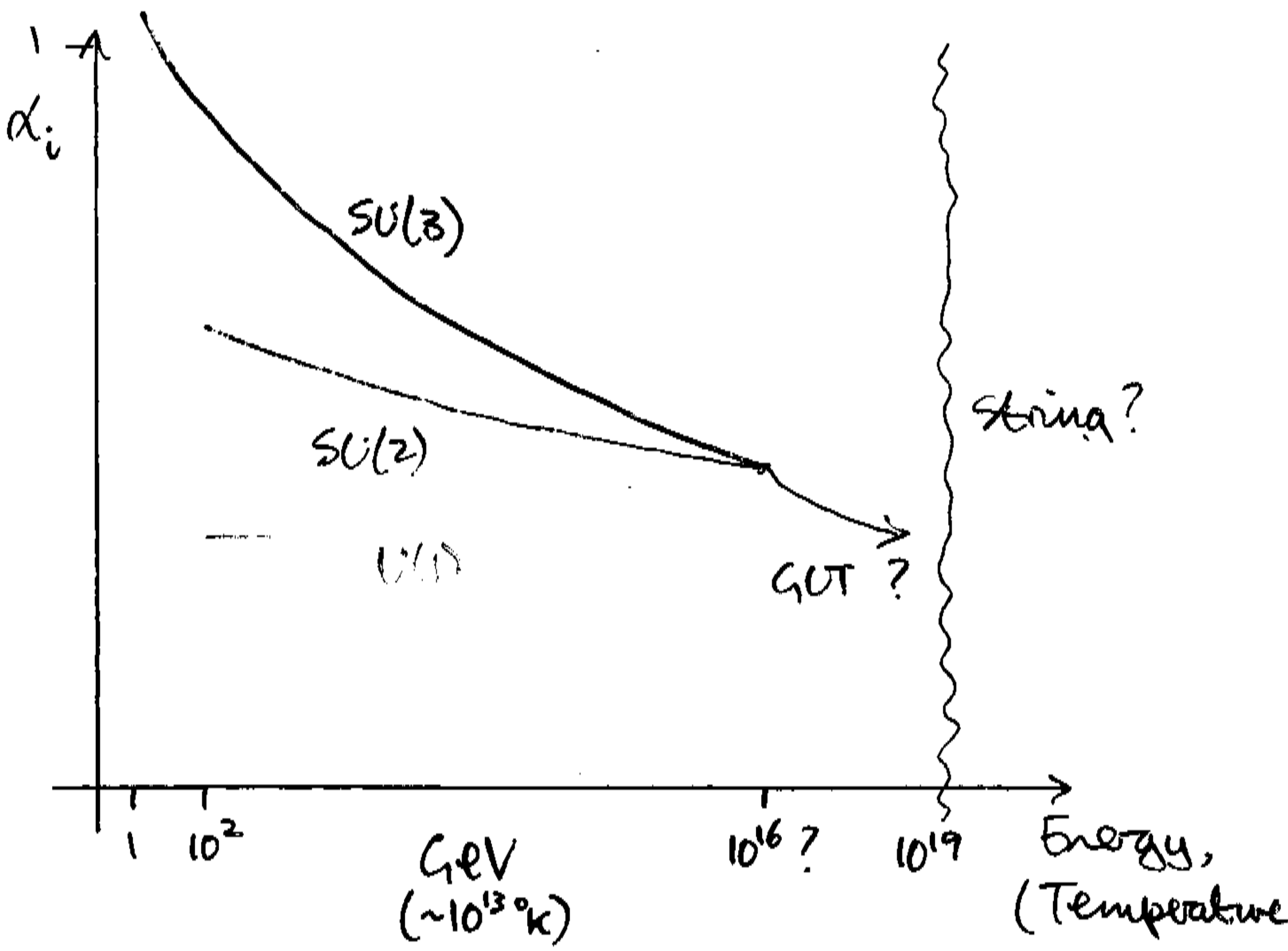
4.2 - La brisure de la supersymétrie due à la supergravité?

4.3 - Vers une "Théorie de Tout"? les supercordes et la théorie M

4.4 - Autres dimensions?

4.5 - L'énergie noire...

1. – Grand Unified Theories



Logarithmic evolution of couplings ⇒

Grand Unification Scale very large:

$$m_X / M_W = \exp\left(O(1)/\alpha_{em} - O(1)/\alpha_{strong} \right)$$

$m_X \simeq 10^{15}$ to 10^{16} GeV

equal couplings at high scale ⇒ predict ratio

$\alpha_1/\alpha_2 \big|_{M_W} \Longleftrightarrow \sin^2\theta_W$ (Georgi + Quinn + Weinberg

simplest model: SU(5) (Georgi + Glashow

possible consequences: proton decay? neutrino masses

Coupling Evolution

$$\frac{dg_i^2}{dt} = b_i \frac{g_i^4}{16\pi^2}$$

$$\sin^2\theta_W(m_Z) = \frac{g'^2}{g^2+g'^2} = \frac{\frac{3}{5}g_1^2(m_Z)}{\frac{3}{5}g_1^2(m_Z)+g_2^2(m_Z)}$$

$$= \frac{1}{1+8x}\left[3x + \frac{\alpha(m_Z)}{\alpha_3(m_Z)}\right]$$

where $x \equiv \frac{1}{5}\left(\frac{b_2-b_3}{b_1-b_2}\right)$

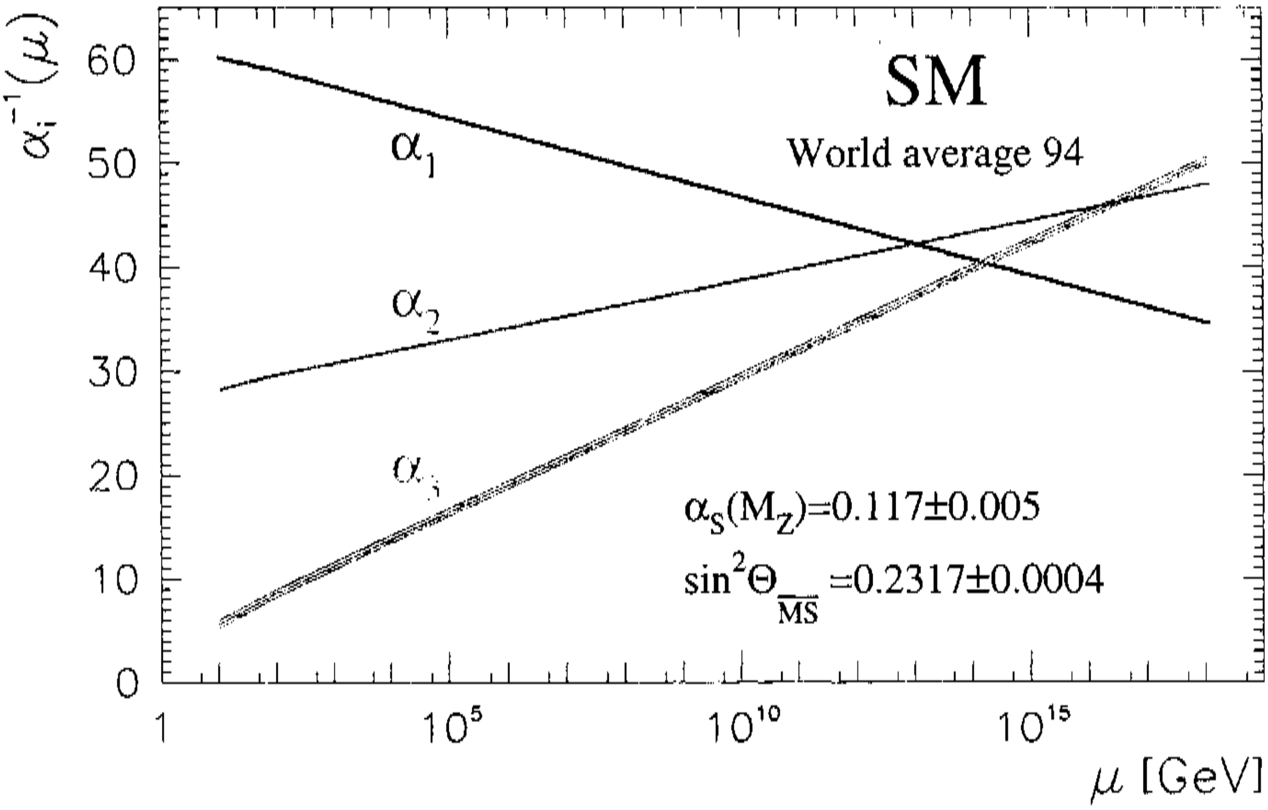
Standard Model		+ supersymmetry	
$\frac{4}{3}N_G - 11$	$\leftarrow b_3 \rightarrow$	$2N_G - 9$	$= -3$
$\frac{1}{6}N_H + \frac{4}{3}N_G - \frac{22}{3}$	$\leftarrow b_2 \rightarrow$	$\frac{1}{2}N_H + 2N_G - 6$	$= +1$
$\frac{1}{10}N_H + \frac{4}{3}N_G$	$\leftarrow b_1 \rightarrow$	$\frac{3}{10}N_H + 2N_G$	$= \frac{33}{5}$
$\frac{23}{218} = 0.1055$	$\leftarrow x \rightarrow$	$1/7$	

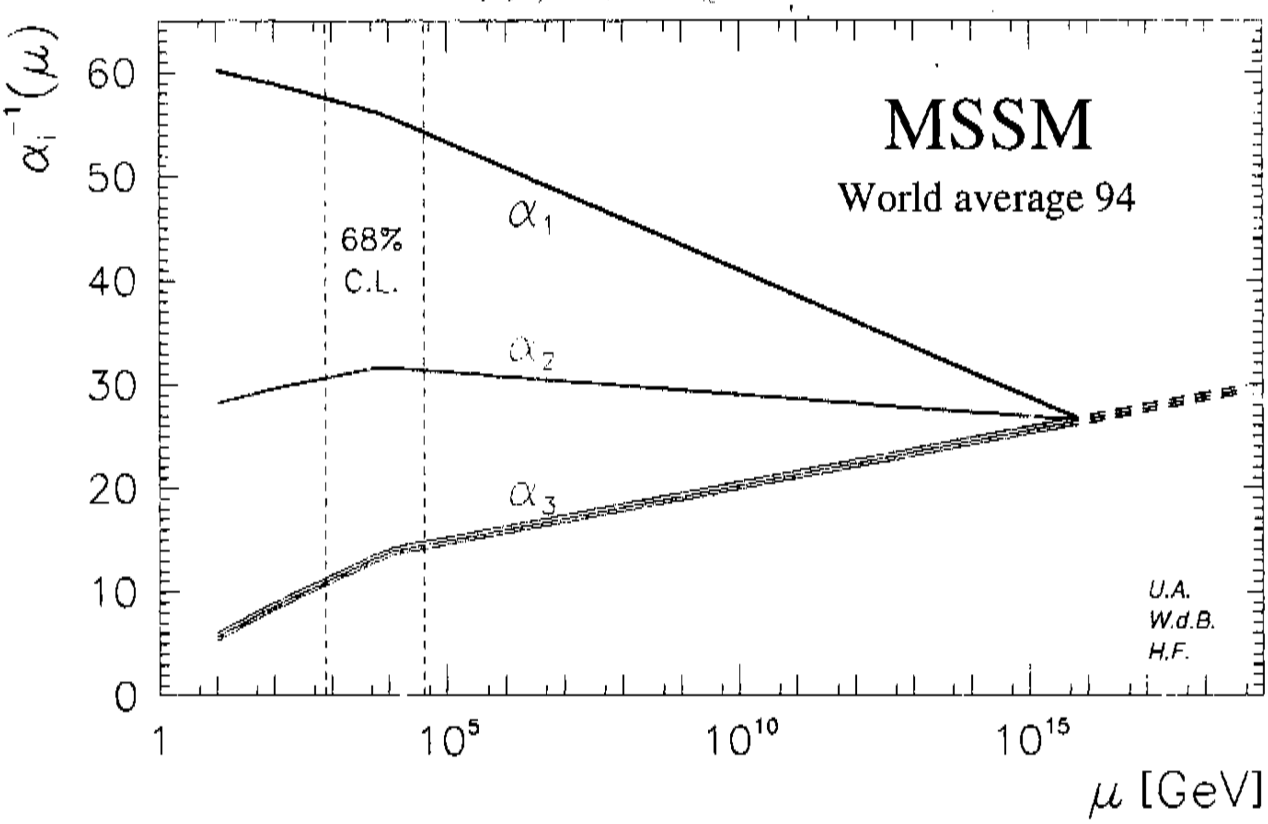
inserting $\alpha(m_Z) = 1/128$, $\alpha_3(m_Z) = 0.119 \pm 0.003$

$$\sin^2\theta_W(m_Z) = 0.2315$$

find $\quad x = 1/6.92 \pm 0.07$

Unification?





1990

S.E. + Kelley + Nanopoulos

Langacker + Luo

Amaldi de Boer Fürstenau

1991

Glasgow HEP Conference 1994 :

$M_S=10^{3.7\pm0.8\pm0.4}$ GeV $\qquad M_U=10^{15.9\pm0.2\pm0.1}$ GeV

return to this later!

Renormalization Group Equations

non-supersymmetric:

$$Q\frac{\partial \alpha_i(Q)}{\partial Q} = -\frac{1}{2\pi}\Big(\underbrace{b_i}_{\text{one-loop}} + \underbrace{\frac{b_{ij}}{4\pi}}_{\text{two-loop}}\alpha_j(Q)\Big)\left[\alpha_i(Q)\right]^2 + \cdots$$

One Loop:

$$b_i = \underbrace{\begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix}}_{\text{gauge}} + N_g \underbrace{\begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix}}_{\text{matter}} + N_H \underbrace{\begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}}_{\text{Higgs}}$$

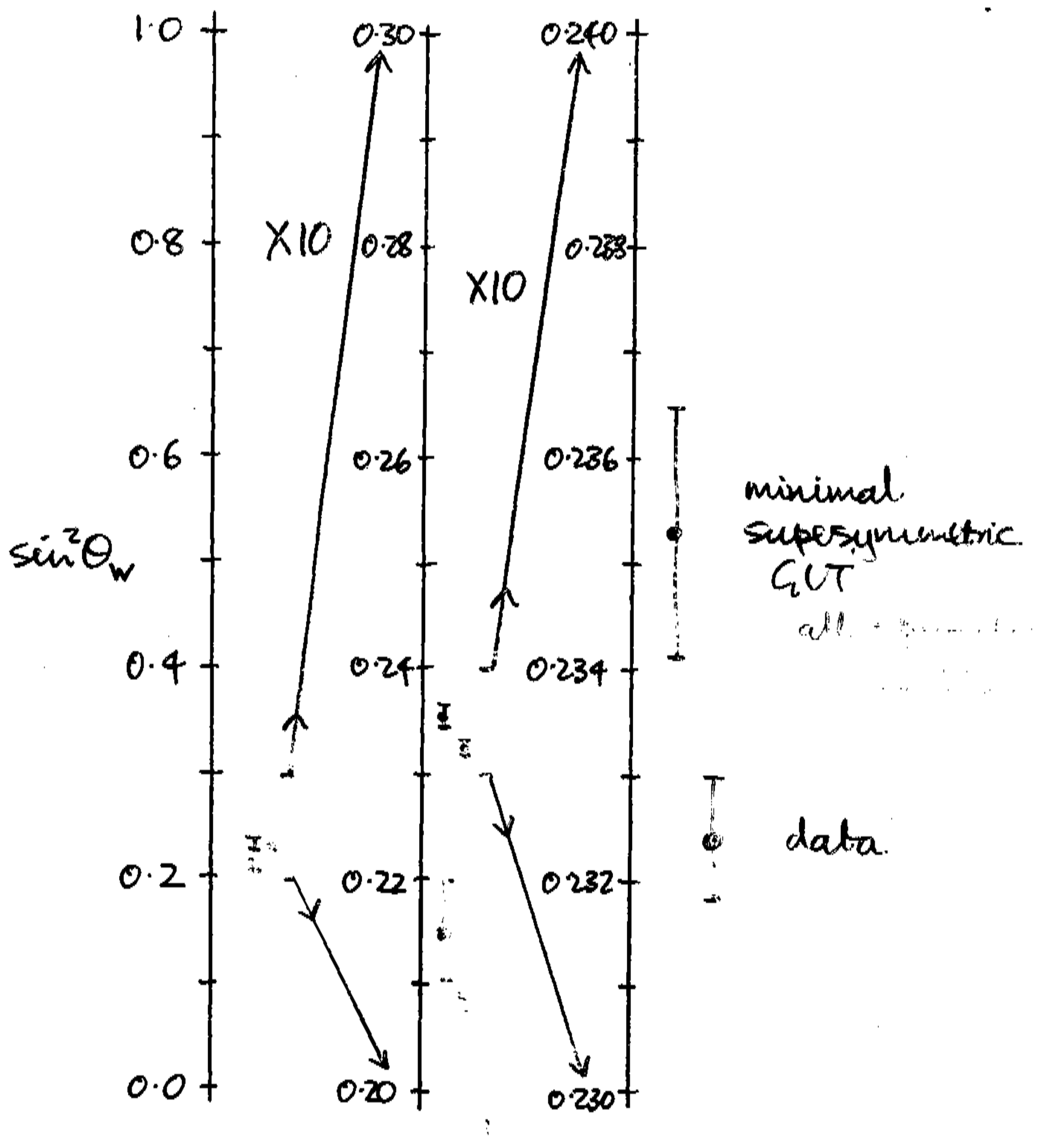
Two loops:

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & -102 \end{pmatrix} + N_g \begin{pmatrix} \frac{19}{15} & \frac{3}{5} & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & \frac{76}{3} \end{pmatrix}$$

$$+ N_H \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 13/6 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

independent of specific GUT model

GUT predictions for $\sin^2\theta_W$



minimal supersymmetric GUT:

$$\sin^2\theta_W(M_Z)_{\overline{MS}} = 0.2317 \pm 0.0003 + (5.4\times10^{-6})(M_H - 100\,\mathrm{GeV}) - (3.03\times10^{-5})(m_t - 165\,\mathrm{GeV}) \ldots$$

Renormalization Group Equations

supersymmetric:

$$Q \frac{\partial \alpha_i(Q)}{\partial Q} = -\frac{1}{2\pi}\left(\underbrace{b_i}_{\text{one-loop}} + \underbrace{\frac{b_{ij}}{4\pi}\alpha_j(Q)}_{\text{two-loop}}\right)[\alpha_i(Q)]^2 + \cdots$$

One-loop:

$$b_i = \underbrace{\begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix}}_{\text{gauge, gaugino}} + N_g \underbrace{\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}}_{\text{(s)matter}} + N_H \underbrace{\begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}}_{\text{Higgs(ino)}}$$

Two-loop:

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + N_g \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ 2/5 & 14 & 8 \\ 11/5 & 3 & \frac{68}{3} \end{pmatrix}$$

$$+ N_H \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 7/2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

independent of specific supersymmetric GUT model

Simple GUT Models

Rules of the game:

1) complex representations for fermions

$\Rightarrow$ can violate charge conjugation: $C: f \to f^c$

needed because weak interactions violate C, P

$\Rightarrow$ f_L and f_L^c in different representations of

$SU(3)_c \times SU(2)_L$: e.g. $\binom{\nu}{e}_L \in (1,2)$, $e_L^c \in (1,1)$ of $SU(3)\times SU(2)$

2) rank of group ≥ 4 $\quad \Rightarrow 2 \quad SU(3)_c + 1$ in $SU(2)_L$

$+1$ in $U(1)_Y$

Simplest example: look at groups of rank 4

$$Sp(8),\ SO(8),\ SO(9),\ F_4,\ SU(3)\times SU(3),\ \underbrace{SU(5)}_{\text{OK!}}$$

useful representations:

vector $\underline{5}$ $\quad F_\alpha$ $\quad$ (conjugate $\underline{\bar{5}}$ F^α)

antisymmetric tensor $\underline{10}$ $\quad T_{[\alpha\beta]}$

adjoint $\underline{24}$ $\quad A^\alpha_\beta \leftarrow$

Simplest GUT model

(Georgi + Glashow

SU(5)

rotations in 5-dimensional complex space

includes Standard Model bosons:

$$
\text{strong interactions: colour } SU(3) \left\{ \left(\begin{array}{ccc:cc} & \text{gluons} & & \bar{X} & \bar{Y} \\ & & & \bar{X} & \bar{Y} \\ & & & \bar{X} & \bar{Y} \\ \hdashline X & X & X & \multicolumn{2}{c}{W\text{'s}} \\ Y & Y & Y & & \end{array} \right) \right\} \text{ weak interactions: } SU(2)
$$

new gauge bosons ⇒ new fundamental forces

embedding of quarks & leptons:

$$
\begin{array}{l} \text{antiquarks} \\ \\ \text{leptons} \end{array}
\begin{pmatrix} \bar{d}_R \\ \bar{d}_Y \\ \bar{d}_B \\ \hdashline e^- \\ \nu_e \end{pmatrix}
\underset{\downarrow}{\overset{\uparrow}{X,Y}}
\left(\begin{array}{ccc:cc} & \bar{u}_{R,Y,B} & & -u_R & -d_R \\ & & & -u_Y & -d_Y \\ & & & -u_B & -d_B \\ \hdashline u_R & u_Y & u_B & \multicolumn{2}{c}{\bar{e}} \\ d_R & d_Y & d_B & & \end{array} \right)
\begin{array}{l} \text{quarks} \\ \\ \leftarrow \text{antilepton} \end{array}
$$

X, Y

$$\bar{q} \underset{X,Y}{\longleftrightarrow} l \qquad q \underset{X,Y}{\longleftrightarrow} \bar{q}$$

<u>Higgs fields</u>: adjoint $\underline{24}$ $\Phi \propto \begin{pmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & -3/2 & \\ & & & & -3/2 \end{pmatrix}$: $SU(5) \to SU(3) \times SU(2) \times U(1)$

$\underline{5}$ $H \propto \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$: $SU(2) \times U(1) \to U(1)$, m_f

Other GUT Groups

Rank 5 with complex representations:

$$SO(10),\ SU(6)$$

matter representations:

$$\underline{16} = \underline{10} + \underline{\bar{5}} + \underline{1} \quad \text{of } SU(5)$$

[illegible]

singlet, "right-handed" ν

adjoint representation: $\underline{45}$

useful for Higgses: $\underline{10}, \underline{16}, \underline{45}, \underline{54}, \underline{120}, \underline{126}$

possible breaking patterns:

$$SO(10) \to SU(5) \times U(1) \to SU(3) \times SU(2) \times U(1)$$

$$SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R \to SU(3) \times SU(2) \times U(1)$$

Rank 6 fashionable in string theory:

$$E_6$$

matter representations:

$$\underline{27} = \underline{16} + \underline{10} + \underline{1} \quad \text{of } SO(10)$$

adjoint representation:

$$\underline{78} = \underline{45} + \underline{16} + \underline{\overline{16}} + \underline{1} \quad \text{of } SO(10)$$

Minimal Supersymmetric SU(5) GUT

GUT multiplets → supermultiplets

matter: $\bar{F} : \underline{\bar{5}}$ $\quad T : \underline{10}$

Higgs: $H, \bar{H} : \underline{5}, \underline{\bar{5}}$ $\quad \Phi : \underline{24}$

electroweak $\quad$ GUT symmetry ✗

$$\langle 0|\bar{H}|0\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix} \qquad \langle 0|\Phi|0\rangle = M \begin{pmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & -3/2 & \\ & & & & -3/2 \end{pmatrix}$$

superpotential: $\quad$ fixed to ↑↑

$$W \ni (\mu + \tfrac{3}{2}\lambda M)\, H\bar{H} + \lambda H \Phi \bar{H} + f(\Phi)$$

cancellation: $\frac{3}{2}M \Leftrightarrow \langle 0|\Phi|0\rangle$

small residual mixing!

requires fine tuning of W: $1/10^{13}$!

BUT technically "natural": no big corrections

$\delta\lambda\,,\ \delta\mu = 0$ $\qquad$ (apart from Z factors)

small μ stays small

But how did it get to be small?

Predictions for quark/lepton mass ratios

in many GUTs, eg. SU(5)

$m_{q,l}$ from $\underline{5}$ Higgses that break $SU(2) \times U(1)$:

$$\mathcal{L}_m = \frac{\lambda}{\sqrt{2}} \bar{F}^a T_{ab} H^{b*} + \frac{\lambda'}{2} \epsilon^{abcde} T_{ab} T_{cd} H_e$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$m_b = m_\tau = \lambda \langle 0 | H^* | 0 \rangle \qquad m_t = \lambda' \langle 0 | H | 0 \rangle, \text{ etc.}$$

Chanowitz + J.E. + Gaillard 1977

2 to 5

2605

$$\Downarrow$$

renormalized after GUT symmetry breaking

$$\frac{m_b}{m_\tau} \simeq \left[\frac{\alpha_3(m_b)}{\alpha_3(m_{GUT})} \right]^{\frac{4}{11 - \frac{4}{3} N_G}}$$

← # generations

(Chanowitz + J.E. + Gaillard 1977

$\simeq 3$ ← before discovery of b quark!

confirmed by higher-order calculations

Result depends on N_G = # generations

$N_G = 4 \Rightarrow m_b > 6$ GeV ✗ (Buras + J.E. + Gaillard + Nanopoulos 1978

Argument to constrain $N_G = 3$ before cosmology, LEP

(Nanopoulos + D. Ross)






Proton Decay

Why not?

no gauge symmetry for baryon # B

$\Rightarrow$ expect $\Delta B \neq 0$ transitions

$\Delta B \neq 0$

processes exist in Standard Model

anomaly $\oplus$ instantons @ $T=0$

sphalerons @ $T\neq 0$

$$\Delta B = N_{generations} = 3$$

Quantum gravity

black holes do not conserve B

$$\tau_p \sim m_P^4/m_N^5 \sim 10^{46}\,y \,?$$

less with supersymmetry

GUTs

generally predict proton decay

Possible $\Delta B \neq 0$ Interactions

constructed from Standard Model fields

$$\bar{u}^c_L \gamma^\mu q_L \, \bar{d}^c_L \gamma_\mu \ell_L \,, \quad \bar{u}^c_L \gamma^\mu q_L \, \bar{e}^c_L \gamma_\mu q_L \quad \leftarrow \text{vector}$$

$$\bar{q}^c_R q_L \; \bar{q}^c_R \ell_L \,, \quad \bar{d}^c_L u_R \; \bar{u}^c_L e_R \quad \leftarrow \text{scalar exchanges}$$

dimension $= 6 \Rightarrow$ coefficient $1/M^2$

$$\tau_p \sim M^4/m_p^5 \quad \leftarrow \text{dimensional analysis}$$

selection rules:

$$\Delta B = \Delta L : \quad p \to \ell^+ X, \bar{\nu} X \qquad p \not\to \ell X$$

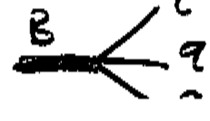
$$\frac{\Delta S}{\Delta B} = 0, -1 : \quad p \to \bar{\nu}\pi^+, \bar{\nu} K^+ \qquad n \not\to e^+ K^-$$

disregarding possible "Cabibbo" suppression

$$\tau(p \to e^+\pi^0) \sim 10^{35}\,\text{y} \times \left(\frac{M}{10^{16}\,\text{GeV}}\right)^4 \times \left(\frac{1/25}{\alpha_{GUT}}\right)^2 \leftarrow \text{coupling strength}$$

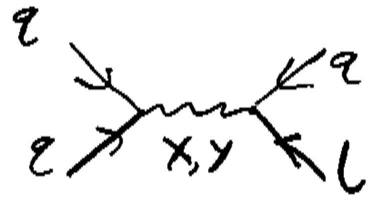
$$\times \left(\frac{0.015\,\text{GeV}^3}{\alpha}\right)^2$$

overlap function in p wave function



Mechanisms in GUTs

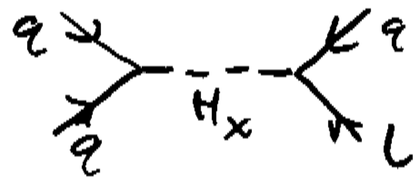
Vector boson exchange



$M = m_{X,Y}$ ~ few $\times 10^{14}$ GeV w/o susy ✗

$p \to e^+ \pi^0, \ldots$ ~ 10^{16} GeV with susy ✓

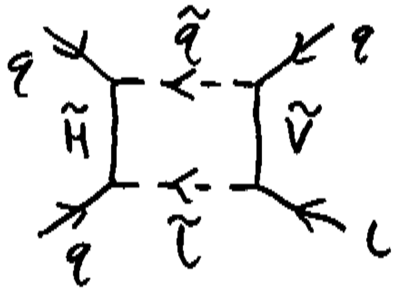
Higgs boson exchange



$M = m_{H_X} \ll m_X$?

$\alpha \sim \left(m_q/m_W\right)^2 \alpha_{GUT} \ll \alpha_{GUT}$ $p \to \mu^+ K^0, \ldots$

Higgsino exchange

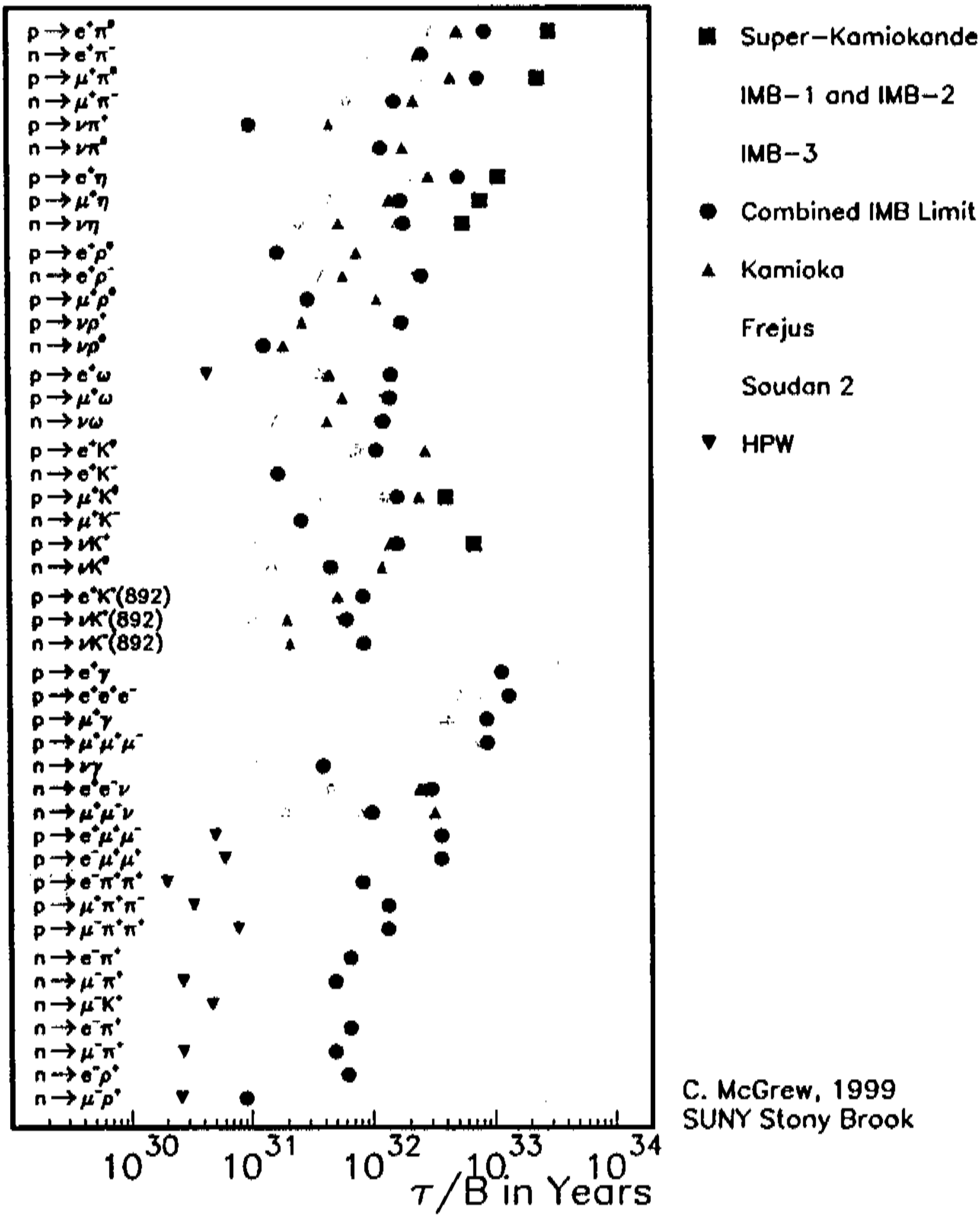


$M^2 = m_{H_X} \tilde{m} \ll m_X^2$?

$\alpha^2 \simeq \alpha_{GUT}^2 \left(\frac{m_q}{m_W}\right)^2 \frac{1}{16\pi^2} f_0(\tilde{m})$ $p \to \bar{\nu} K^+, \ldots$

probably X in minimal supersymmetric GUTs

Nucleon Lifetime Limits



http //superk.physics.sunysb.edu/ mcgrew/pdk limits.ps

4.2 - Supersymmetry Breaking

is necessary: $m_{\tilde{e}} \neq m_e$, $m_{\tilde{\gamma}} \neq m_\gamma$, ...

is it explicit or spontaneous ?

$$\langle 0|Q|\chi\rangle = f_\chi^2 \neq 0$$

BUT in global supersymmetry

spontaneous susy breaking $\Rightarrow$ vacuum energy $E_0 > 0$

$$\{Q, Q\} \propto \gamma_\mu P^\mu$$

take vacuum expectation value:

$$\langle 0| \quad |0\rangle \qquad \langle 0| \quad |0\rangle$$

find: $|\langle 0|Q|\chi\rangle|^2 = f_\chi^4 \propto \langle 0|P_0|0\rangle \equiv E_0$

spontaneous susy X $\quad f_\chi^4 = E_0 > 0$

how to get $E_0 > 0$?

$$V = \frac{1}{2}\sum_i \left|\frac{\partial W}{\partial \phi^i}\right|^2 + \frac{1}{2}\sum_\alpha g_\alpha^2 |\phi^* T^\alpha \phi|^2$$

either $F > 0$ or $D > 0$

How to break global supersymmetry?

$$V = \sum_i |F_i|^2 + \frac{1}{2}\sum_\alpha g_\alpha^2 D_\alpha^2$$

either $F > 0$ or $D > 0$

(O'Raifeartaigh) (Fayet + Iliopoulos)

- needs additional chiral (matter) fields
- "artificial" couplings

- needs extra $U(1)$ gauge group
- many new fields

simplest example:

$$W = \alpha\, A B^2 + \beta\, C(B^2 - m^2)$$

$$F_A = \alpha B^2, \quad F_B = 2B(\alpha A + \beta C), \quad F_C = \beta(B^2 - m^2)$$

effective potential:

$$V_F = \sum_i |F_i|^2$$

$$= |2B(\alpha A + \beta C)|^2 + |\alpha B^2|^2 + |\beta(B^2 - m^2)|^2$$

cannot vanish simultaneously

$\Rightarrow V > 0 \Rightarrow$ susy $\times$

Ugly! try local supersymmetry (supergravity)

Simplest example of D breaking

U(1) gauge theory, one chiral multiplet

$$V = \frac{1}{2}\left(\xi + g A^* A\right)^2$$

not allowed for non-Abelian theory

unit charge

minimum of potential: $\langle A \rangle = 0$, $V = \frac{1}{2}\xi^2$

supersymmetry broken: $\neq 0$

easy to check

$$m_A = g\xi \ , \quad m_\psi = 0 \ , \quad m_V = m_{\tilde{V}} = 0$$

applicable to U(1) of Standard Model?

NO: fields have $\pm$ charges

$\Rightarrow$ minimum always @ $V = 0 \Rightarrow$ susy ✓

has unphysical $\langle 0|\phi|0\rangle \neq 0$

Local Supersymmetry and susy breaking

$\equiv$ supergravity

Why?

make supersymmetry local $\quad E \to E(x)$

like gauge theories

elegant mechanism for supersymmetry breaking

super-Higgs mechanism

unify all interactions

$G(J=2) \to \tilde{G}(J=\frac{3}{2}) \to V(J=1) \to \ldots$ in $(N>1)$ supergravity

gravity and supersymmetry both exist

✓ ? !

cosmological constant

to discuss, need consistent coupling of

$(N=1)$ supersymmetric matter to gravity

What?

introduce gravitino $\quad \tilde{G}$: spin $\frac{3}{2}$

gauge fermion

graviton supermultiplet $\begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix} \begin{matrix} G \\ \tilde{G} \end{matrix}$

Local Supersymmetry: Supergravity

make supersymmetry transformation local

$$\epsilon(x)$$

transformations for chiral smultiplet:

$$\delta_i \phi = (\sqrt{2}\,\bar{\epsilon}_i)\psi$$

$$\delta_j \psi = -i\not{\partial}\phi(\sqrt{2}\,\epsilon_j) + \cdots$$

combination: $[\delta_i, \delta_j](\text{field}) = -2(\bar{\epsilon}_j \gamma_\mu \epsilon_i) i\partial_\mu(\text{field})$

translation: $i\partial_\mu \Leftrightarrow P_\mu$

ϵ_i independent of x $\Rightarrow$ global translation

dependent on x $\Rightarrow$ local coordinate transform

gauge field? gravity?

Supergravity

consider simplest supersymmetric model:

$$L = i\bar{\psi}\not{\partial}\psi + |\partial_\mu \phi|^2$$

$$\delta\psi = -i\not{\partial}\phi\,\epsilon(x) \qquad \delta\phi = \bar{\epsilon}(x)\psi$$

$$\Rightarrow \delta L = \partial_\mu(\cdots\cdots) + 2\bar{\psi}\gamma_\mu\not{\partial}\phi\,\partial^\mu\epsilon(x) + \text{h.c.}$$

must cancel this: need more fields

Analogy with Gauge Theories

$$\delta(i\bar{\psi}\not{\partial}\psi)$$

Gauge | Susy

$\psi \to e^{i\epsilon(x)}\psi$ | $\mathcal{E}(x)$

$-\bar{\psi}\gamma_\mu\psi\,\partial^\mu\epsilon(x)$ variation $\cdots + 2\bar{\psi}\gamma_\mu\gamma_5\,\partial^\mu\mathcal{E}(x)$

cancelled by adding

$$\begin{cases} \bar{\psi}\gamma_\mu\psi A^\mu(x) \\ \delta A_\mu(x) = \partial_\mu\epsilon(x) \end{cases}$$

$\kappa\bar{\psi}\gamma_\mu\not{\gamma}_5\psi^\mu(x)$

$\delta\psi_\mu = -\frac{2}{\kappa}\partial_\mu\mathcal{E}(x)$

Gauge Boson | Gauge Fermion $\equiv$ gravitino

spin $= 3/2$

Graviton $\oplus$ Gravitino Lagrangian:

$$L = -\frac{1}{2\kappa^2}\sqrt{-g}\,R \quad - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma$$

$g \equiv \det(g_{\mu\nu})$ — curvature — covariant derivative.

$g_{\mu\nu} \equiv e^m_\mu e^n_\nu \eta_{mn}$

Minkowski

$\mathcal{D}_\rho = \partial_\rho + \frac{1}{4}\omega_\rho^{mn}[\gamma_m, \gamma_n]$

$\omega_\rho^{mn} \equiv$ spin connection

is invariant under local susy transform^n:

$$\delta e^m_\mu = \frac{\kappa}{2}\bar{\mathcal{E}}(x)\gamma^m\psi_\mu(x)$$

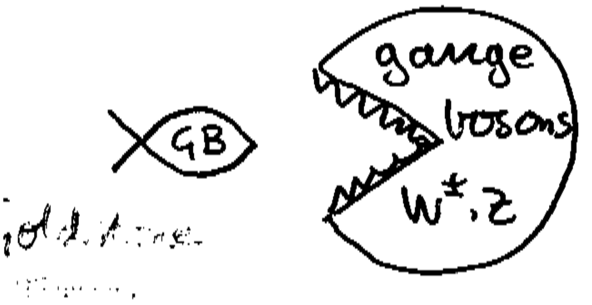
$$\delta\omega^{mn}_\mu = 0$$

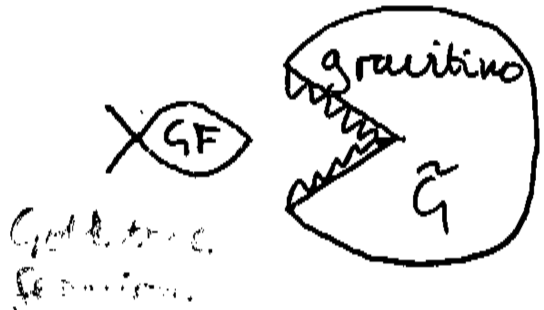
$$\delta\psi_\mu = \frac{1}{\kappa}\mathcal{D}_\mu\epsilon(x)$$

Supersymmetry breaking

SuperHiggs mechanism

analogy with gauge symmetry breaking





(Polonyi
(Cremmer et al

$\Rightarrow \quad m_{W^{\pm}, Z^0} \neq 0 \qquad\qquad \Rightarrow \quad m_{\tilde{G}} \neq 0$

count polarization states

$\underset{m=0}{W(2)} \oplus GB(1) = \underset{m\neq 0}{W(3)} \qquad \underset{m=0}{\tilde{G}(2)} \oplus GF(2) = \underset{m\neq 0}{\tilde{G}(4)}$

massive $J=1$ $\qquad\qquad$ massive $J=3/2$

spontaneous breakdown of local susy:

graviton mass: $m_G = 0 \neq$ gravitino mass $m_{\tilde{G}} \neq 0$

only known consistent way of breaking local supersymmetry

can be achieved with zero vacuum energy

$\langle 0|V|0\rangle = 0 \iff \Lambda = 0$

Coupling Supergravity to Matter

Gravitino mass: $m_{3/2} \; (\equiv m_{\tilde{G}})$

Gaugino masses: $m_{1/2} \propto m_{3/2}$

universality not a problem

Scalar masses: $m_0 \propto m_{3/2}$

no obvious reason for universality
e.g.: broken in string models

Trilinear scalar couplings: $A_\lambda \lambda \phi^3$: $A_\lambda \propto m_{3/2}$

again non-universal?

Bilinear scalar coupling: $B_\mu \mu \bar{H} H$: $B_\mu \propto m_{3/2}$

only one in MSSM

Soft menagerie:

$$- \sum_a m_{1/2_a} \tilde{V}_a \tilde{V}_a - \sum_i m_{0_i}^2 |\phi_i|^2 - \left(\sum_\lambda A_\lambda \lambda \phi^3 + \text{herm. conj.} \right)$$
$$- \left(B_\mu \mu \bar{H} H + \text{herm. conj.} \right)$$

many parameters + phases

Effective Low-Energy Theory

with softly-broken supersymmetry

$$\underbrace{m_0 \quad , \quad m_{1/2} \quad , \quad A}_{} \qquad (B)$$

parameters renormalized analogously to g_a

gaugino masses

same renormalization as α_i @ 1 loop:

$$M_a = \frac{\alpha_a}{\alpha_{GUT}} \cdot m_{1/2}$$

assume universal (?) input @ M_{GUT}

scalar masses

$$\frac{dm_{0_i}^2}{dt} = \frac{1}{16\pi^2}\left[\lambda^2 (m_0^2 + A_\lambda^2) - g_a^2 M_a^2 \right]$$

group-theoretical coefficients

negligible for first two generations:

$$m_{0_i}^2 = m_0^2 + C_i \, m_{1/2}^2$$

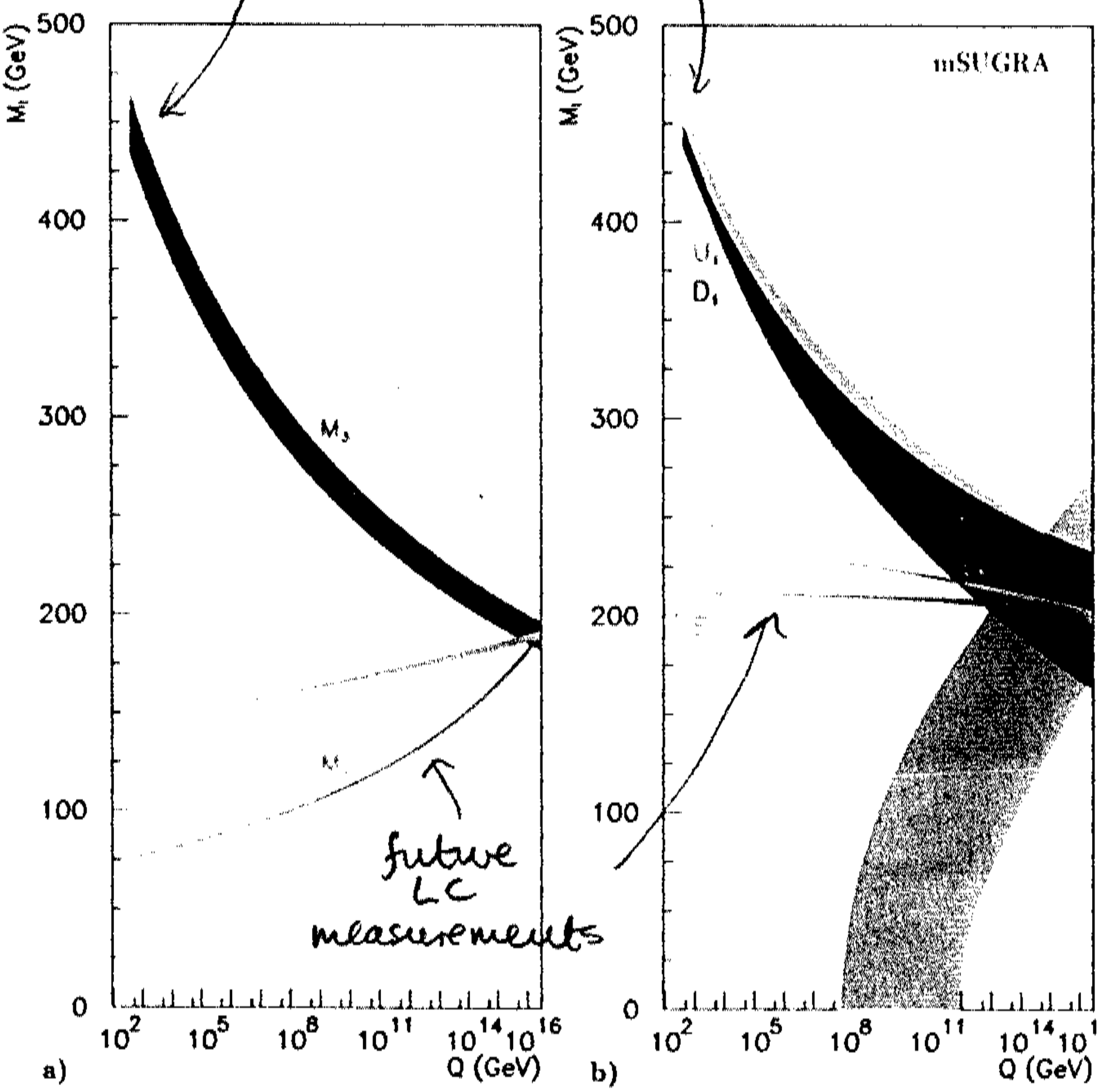
calculable

important for third generation, Higgs

Will Spartide Masses Unify?

mSUGRA vs GMSB vs ...

measured @ LHC



future LC measurements

Figure 1: **mSUGRA:** *Evolution of (a) gaugino and (b) sfermion mass parameters in the bottom-up approach. The mSUGRA point probed is characterized by the parameters M_0 = 200 GeV, $M_{1/2}$ = 190 GeV, A_0 = 550 GeV,* $\tan\beta = 30$, *and* $\mathrm{sign}(\mu) = (-)$. *[The widths of the bands indicate the 95% CL.]*

(Blair + Porod + Zerwas

3 Towards a Theory of Everything?

including gravity

- unify all the fundamental interactions
- solve the problems of quantum gravity

terrible infinite results of calculations

Possible answer: particles → extended objects

first incarnation:

string theory:

requires extra dimensions and/or interactions

4 + 6 = Calabi-Yau, or...?

new incarnation:

M theory: include other extended objects +1

membranes, solids, ...

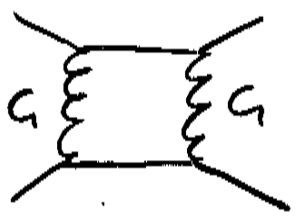
How to test these ideas?

Problems of Quantum Gravity

How to make a calculable (renormalizable) theory?

Couplings have dimensions $(G_N = 1/m_P^2)^n$

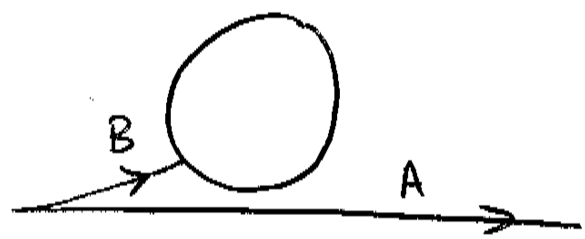
$\Rightarrow$ loop corrections



are generally infinite: $(G_N \Lambda^2)^n \leftarrow$ infinite series

How to treat non-trivial space-time backgrounds

e.g. black holes:



$\rightarrow$ pure states may evolve to $|B,A\rangle\langle A,B| \longrightarrow \sum_i |A\rangle_i \, {}_i\langle A|$

mixed states:

conflict with conventional quantum mechani

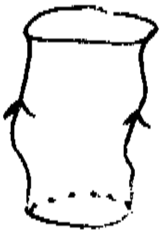
Introduction to String Theory

- point-like particles → extended objects

  soften divergences @ short distance

- initially: 

  open + closed strings

- worldline  → worldsheet 

- dynamics characterized by 2-dimensional field theory ← generically finite

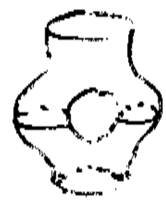
- conformal field theory:

  T  T = conformal anomaly $c = 26$

  superstring: $c = \boxed{10} \times \frac{3}{2}$

corresponds to classical Lorentz-invariant vac^m

interactions  → plumbing 

sum over worldsheets:  +  + ⋯

Classes of String Model

Bosonic

needs $d=26$ for consistent quantum formulation, no fermions, flat space-time unstable

Super

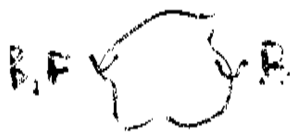
needs $d=10$, has fermions, flat space-time stable, fermions not chiral: $f_L \equiv f_R$

Heterotic

needs $d=10$, has chiral fermions

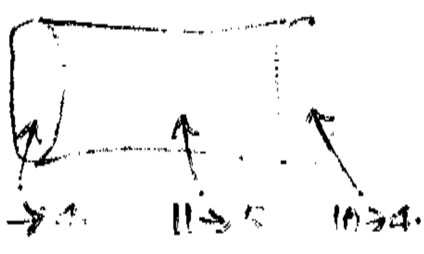
$d=26$ bosons $\Rightarrow$ gauge group G in $d=10$, flat space-time stable

d=4 Heterotic



obtained by compactifying $d=10$, or directly in $d=4$, large choice of G, flat space-time, chiral

M theory



formulated in $d=11$: boundary $d=10$

compactify: $d=11 \to d=5$, $d=10 \to d=4$

$\simeq d=11$ SUGRA $\quad L_5 \gg L_{10}$?

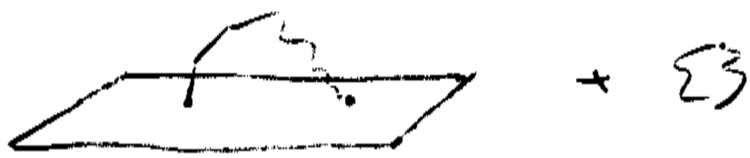
Non-Perturbative String Theory

- higher-dimensional extended objects:

  string solitons : D branes

  "

  balls (walls) of string: 

- open strings ending on membranes
- change in critical dimension:

  superstring: $d=10 \longrightarrow d=\boxed{11}$

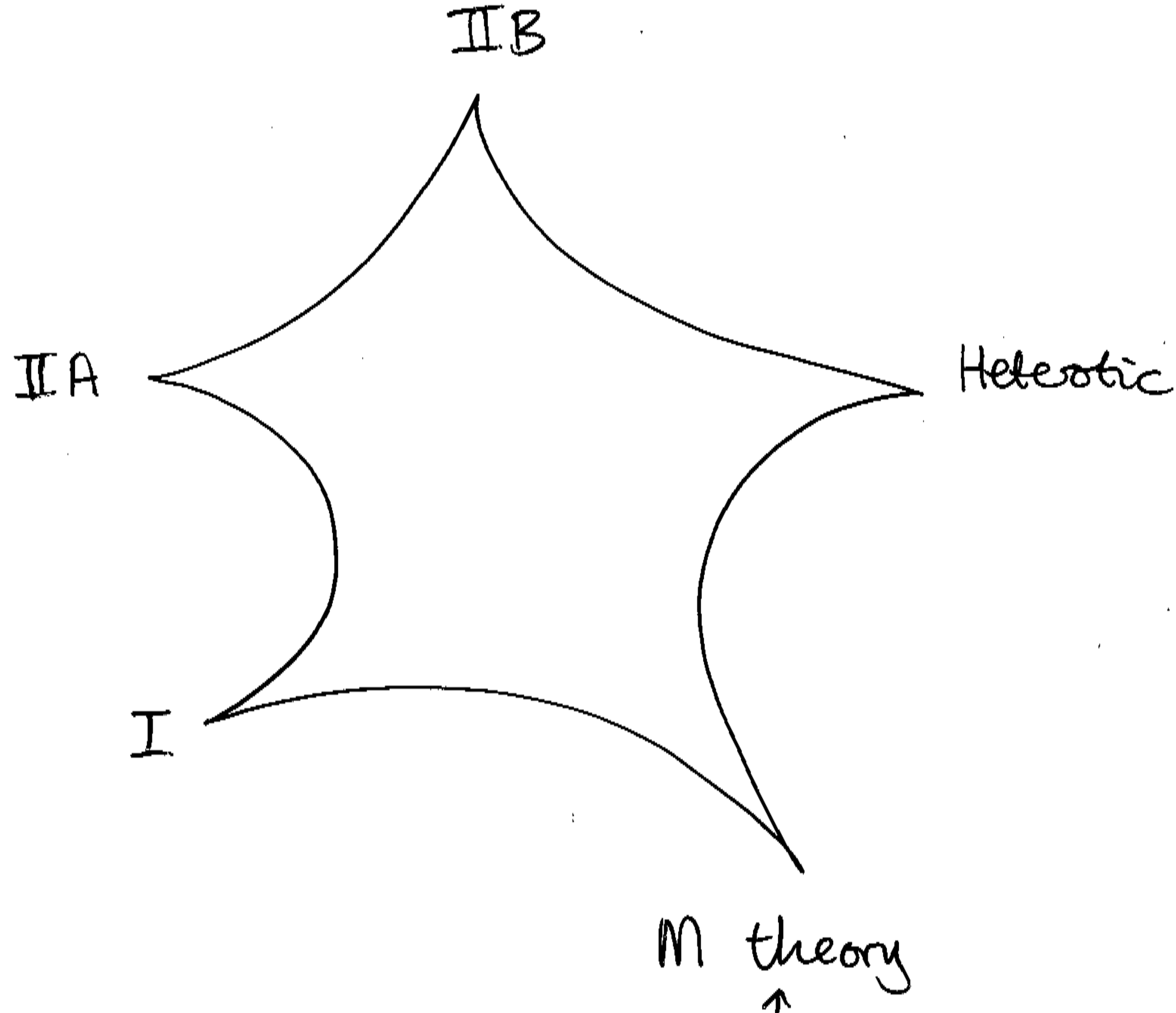
- extra dimension large $\Leftrightarrow$ strong coupling

model black holes as D branes

entropy $S=\frac{1}{4}A \Leftrightarrow$ # quantum states

Chart of String Dualities

all string models limits of same theory



11th dimension related to coupling strength

$m_{GUT} \ll m_P \longleftrightarrow$ one "large" dimension

Ambiguities in String Compactifications

- There are literally billions of Calabi-Yau spaces
- Number of generations, other particles related to their topological properties

  many choices with $N_g = 3$!

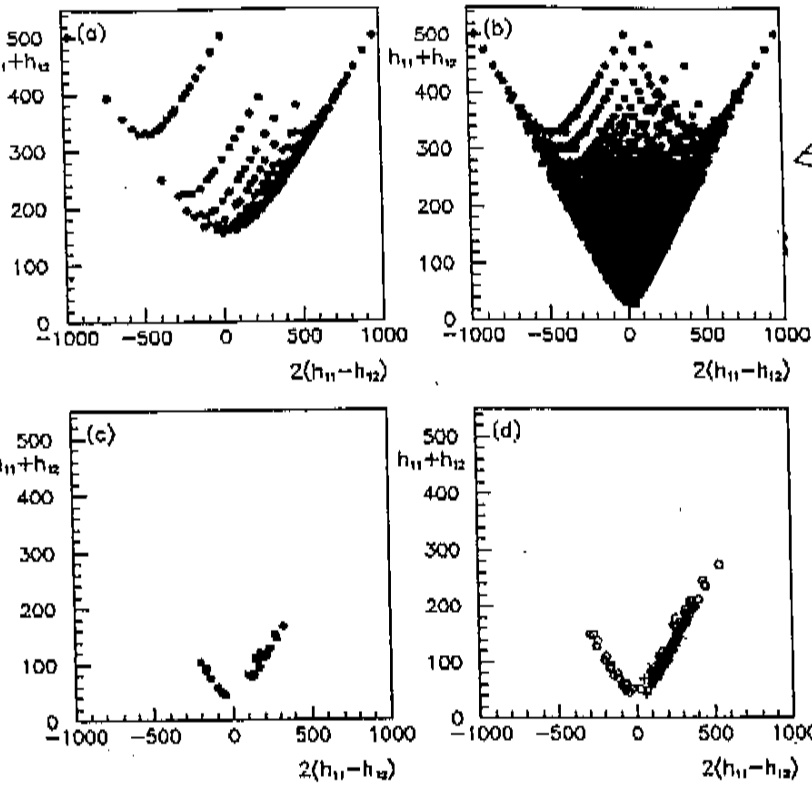
- Each Calabi-Yau space can be deformed,

parametrized by moduli fields

arbitrary values of moduli!

Will these options be determined by non-perturbative effects in string theory?

Or could one imagine different "Universes" with different # dimensions, # generations ...?



← 174,767 Calabi-Yau

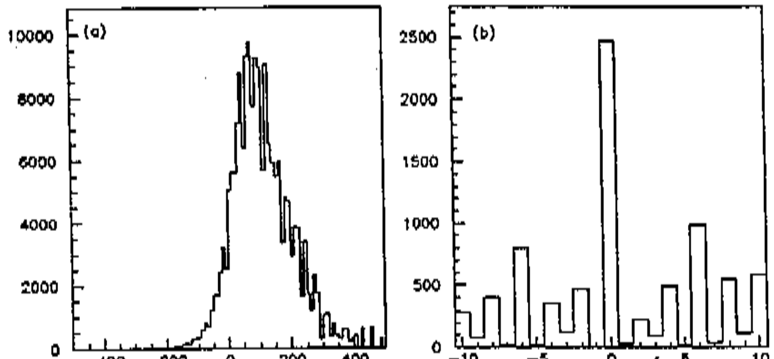
a few more →

← and a few more

$= 2N_g$

Figure 2: *Scatter plots of the Hodge numbers (a) for all the 542 CY_3 spaces in one of the 4 242 two-vector chains, namely $m(0,1,6,14,21) + n(1,0,6,14,21)$, (b) for all the CY_3 spaces constructed via two-vector chains, (c) for the addit ional CY_3 spaces, not obtainable from any of the two-vector chains, that are fo und in the three-vector chain $m(0,0,1,1,1) + n(0,1,0,1,2) + l(1,0,0,1,2)$, and (d) for all the additional CY_3 spaces constructed via chains obtained from three (circles) four (crosses) and five vectors (plus signs).*

histogram of # of generations →



$= N_g$

Figure 3: *Histograms of the CY_3 spaces constructed via two-vector chains, as functions of $h_{11} - h_{12}$ and hence the number of generations $N_g = |h_{11} - h_{12}|$, (a) for all the CY_3 spaces found, and (b) for the CY_3 spaces and mirrors with $N_g \leq 10$.*

211 spaces with 3 generations

Sampling of Calabbi-Yau Models found with simple algebraic construction

Predictions of String Theory

- Upper limit on # of dimensions:

  $d \leq 26$, $d \leq 11$ if supersymmetry

  previously any number of dimensions would have been allowed

- Upper limit on size of gauge group:

  $G \subset E_6$? rank $G \leq 22$?

  if Calabi-Yau could be bigger with non-perturbative effects

- Upper limit on matter representations:

  no adjoints such as $\underline{3}$ of SU(2), $\underline{24}$ of SU(5) except in complicated (unrealistic?) models

- String unification scale calculable:

  $$m_U = g \times 3 \times 10^{17}\,\text{GeV} \times f(\text{moduli}, \ldots)$$

  below m_P, close to GUT scale

- Prediction for m_t

generic feature of string models:

$$\lambda = \mathcal{O}(1) \times g$$

moduli, ... gauge coupling

renormalization down to weak scale

$$\Rightarrow \quad m_t \lesssim 200 \text{ GeV}$$

what about lighter quarks and leptons?

higher-order terms in superpotential.

$$\mathcal{O}(1) \, \frac{g^{n-2}}{m_P^{n-3}} \, \Phi^n \qquad \lambda_{eff} = \mathcal{O}(1) \, g^{n-2} \left(\frac{\langle 0|\Phi|0\rangle}{m_P} \right)^{n-3}$$

calculable in specific string models

- Options for supersymmetry breaking

$$\underbrace{\langle F_{hidden} \rangle \neq 0 \qquad \langle F_{dilaton} \rangle \neq 0}_{\text{universal susy } \times} \qquad \underbrace{\langle F_{modulus} \rangle \neq 0}_{\text{non-universal, small } m_{1/2}\text{?}}$$

4 Extra Dimensions?

suggested by string theory: 6? 7?

originally thought to be small: 10^{-33} cm $\sim 1/10^{19}$ GeV

help unify gauge with gravity: 10^{-29} cm $\sim 1/10^{15}$ GeV

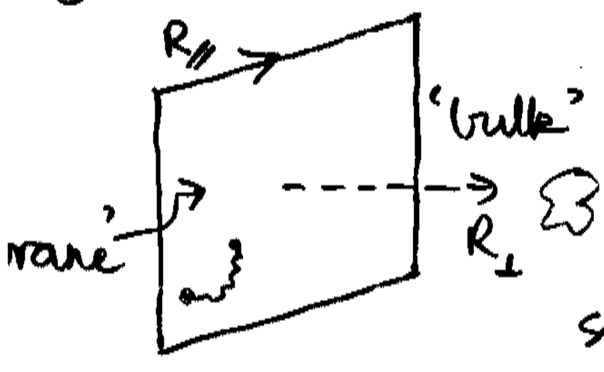
break supersymmetry : 10^{-17} cm $\sim 1/1$ TeV

~~abolish~~ rewrite hierarchy: 10^{-2} cm $\sim 1/10^{-3}$ eV

2 dimensions : $m_s \sim 1$ TeV

string scale

<u>general framework:</u>



$$m_P^2 = \frac{1}{g_s\, g_{YM}^2} M_s^{2+n} R_{\perp}^n$$

$g_s \lesssim O(1)$, $g_{YM} = O(1)$

string coupling

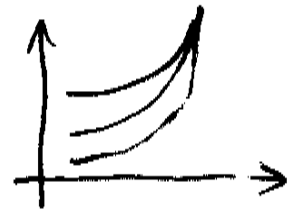
extra dimensions felt by gravity?

$n=2$: limits from graviton emission : SNae $\Rightarrow R \lesssim 10^{-5}$ cm

Cosmology $\Rightarrow R \lesssim 10^{-8}$ cm

felt by matter particles?

is coupling unification natural?



Signature of Kaluza-Klein excitations @ LHC

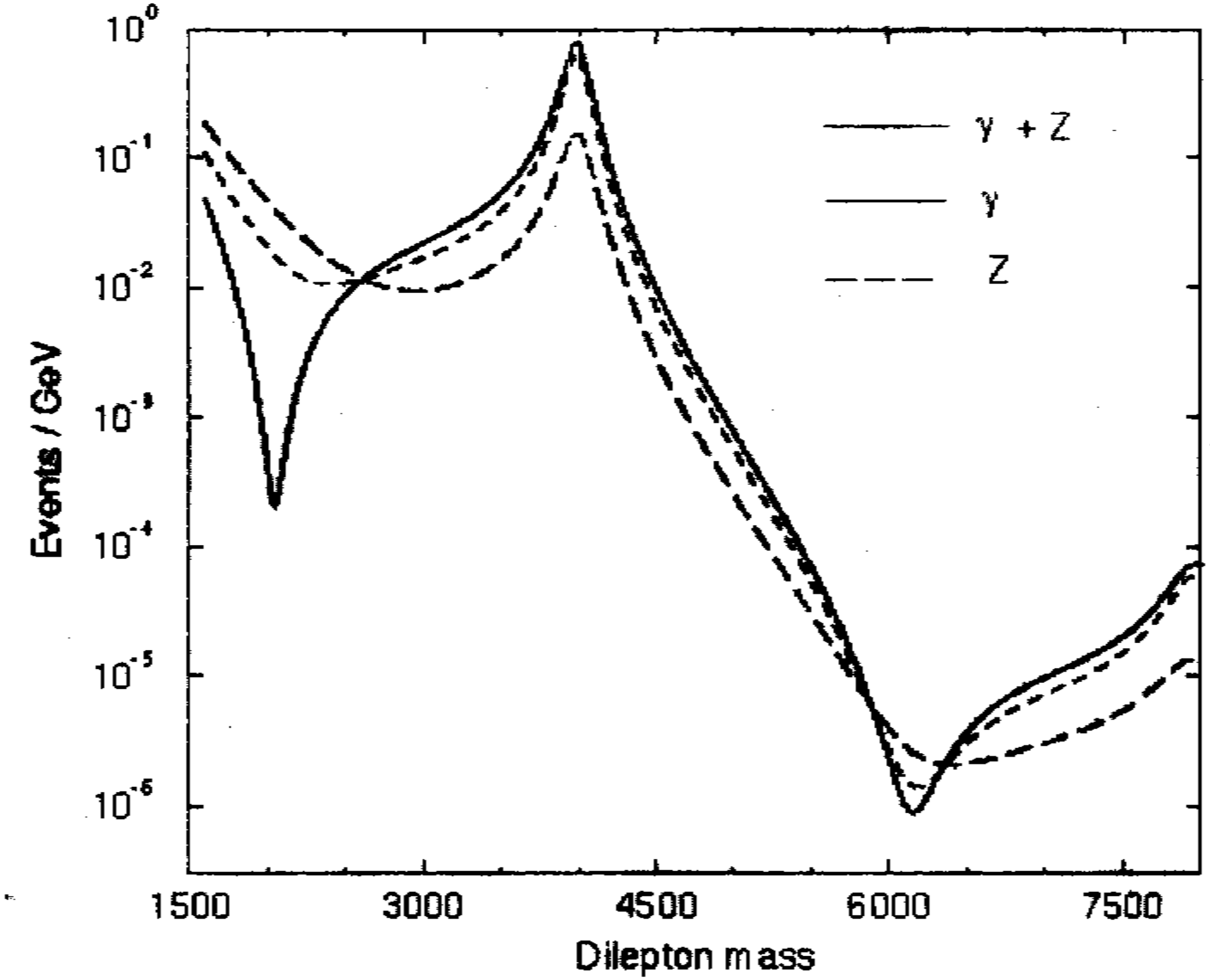
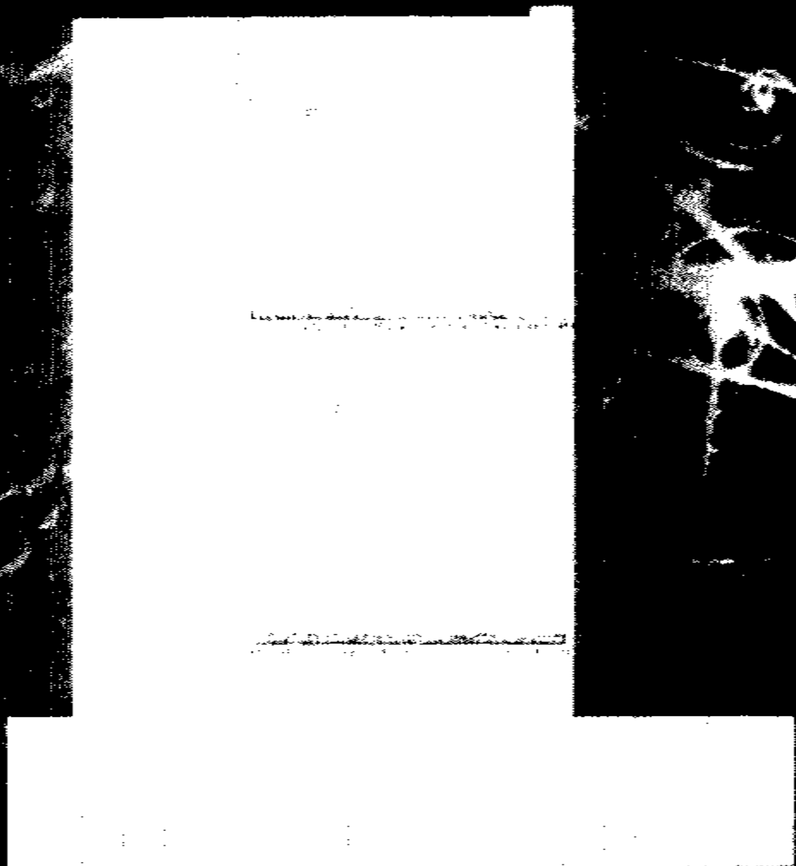
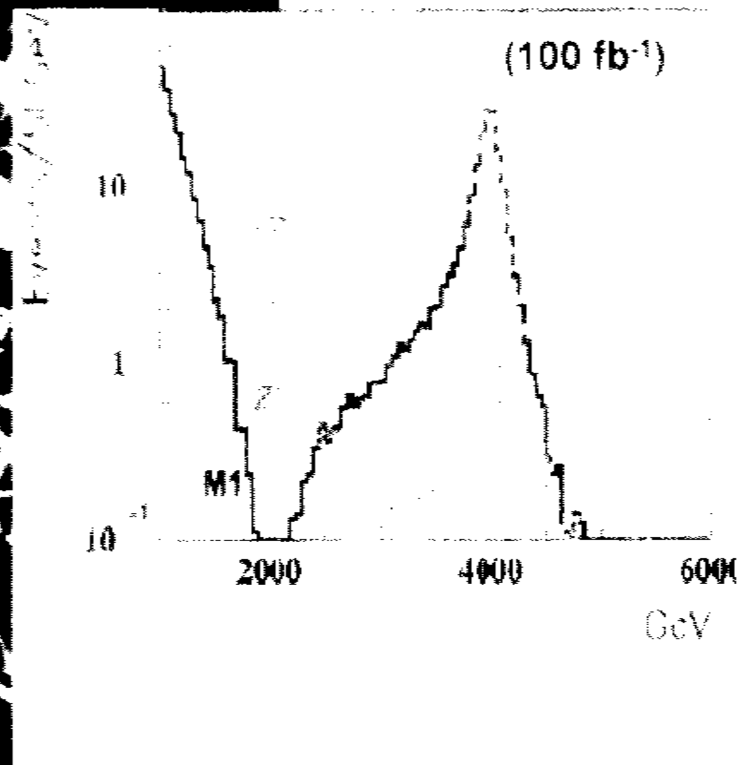


Figure 6: *First resonances in the LHC experiment due to a KK excitation of photon and Z for one extra-dimension at 4 TeV. From highest to lowest: excitation of photon+Z, photon and Z boson.*

If gravity feels them …



If gauge interactions feel them …



KK towers

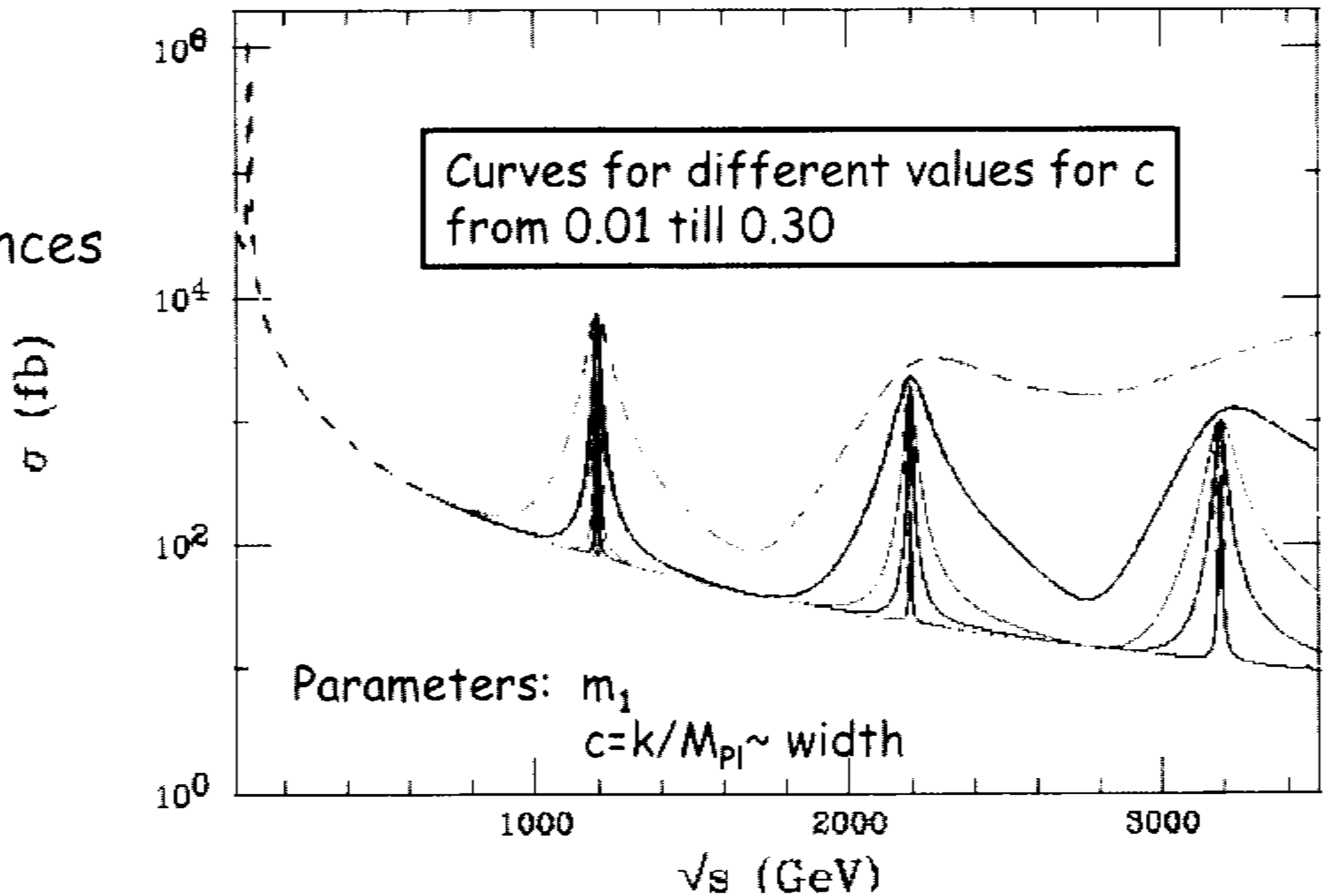
Extra Dimensions Randall-Sundrum phenomenology (curves by T. Rizzo)

SM fields on brane and graviton in bulk

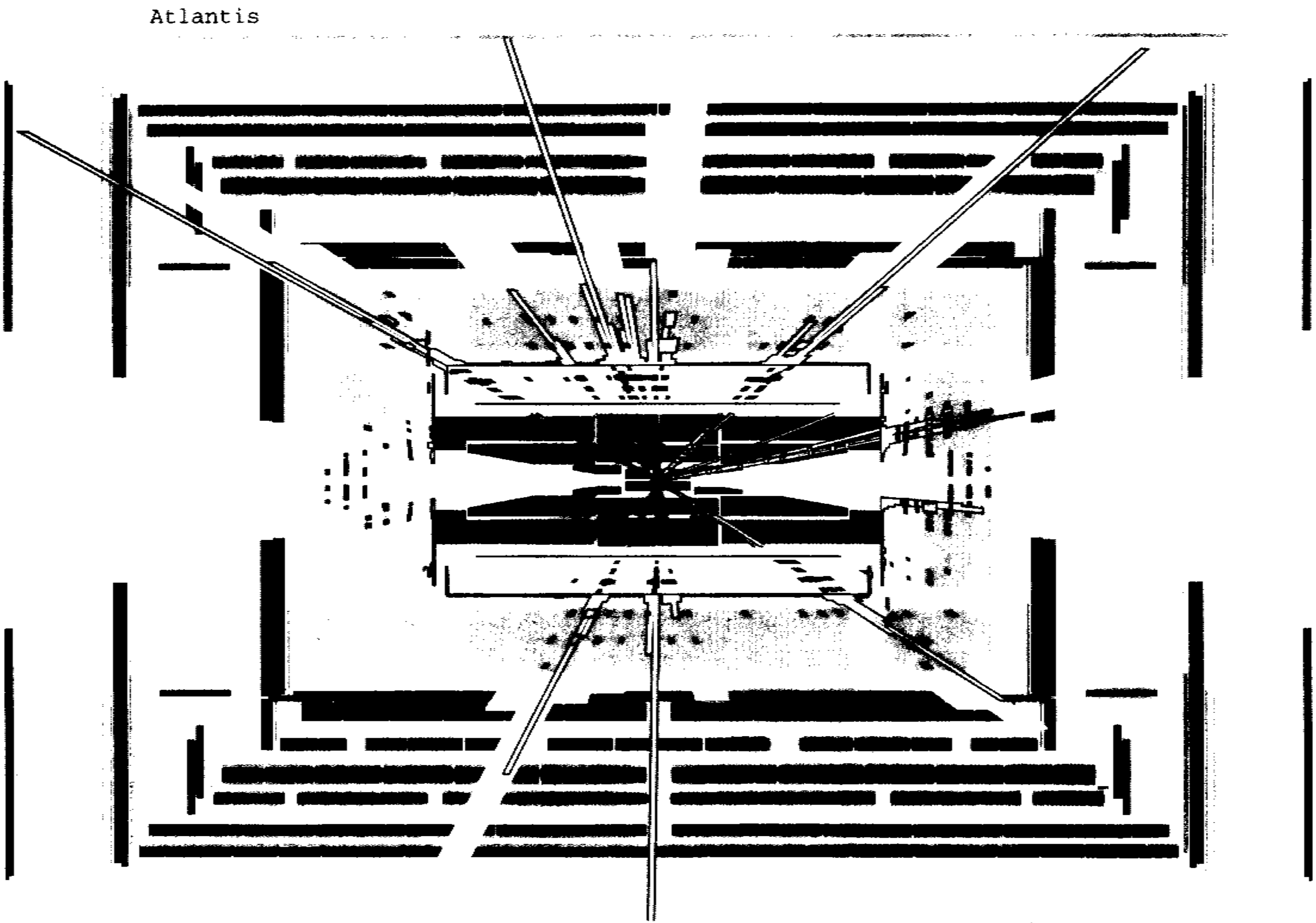
Observe KK resonances in e.g. $e^+e^- \to \mu\mu$ cross sections

LC is like a KK factory

Allows to measure properties of KKs (spin, BRs...)



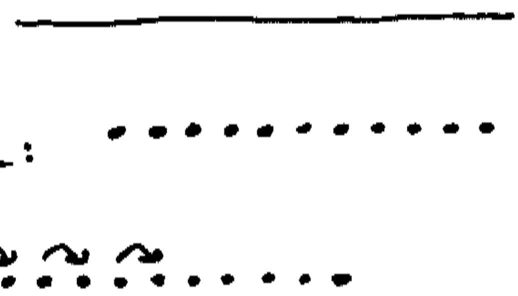
Can determine parameters c up to 0.2%, M better than 0.1%


Atlantis

if you want something wilder ...

Fewer dimensions @ higher energies?

think of dimension
as low-E approx^n to lattice:
motion $\Leftrightarrow$ hopping.



Consider theory: hopping interaction $\Rightarrow$ @ high E
no hopping $\Rightarrow$ no motion ?
way to reduce # of dimensions

Non-commutative field theories?

$$[x_i , x_j] = \theta_{ij} \neq 0$$

cf quantum Hall effect

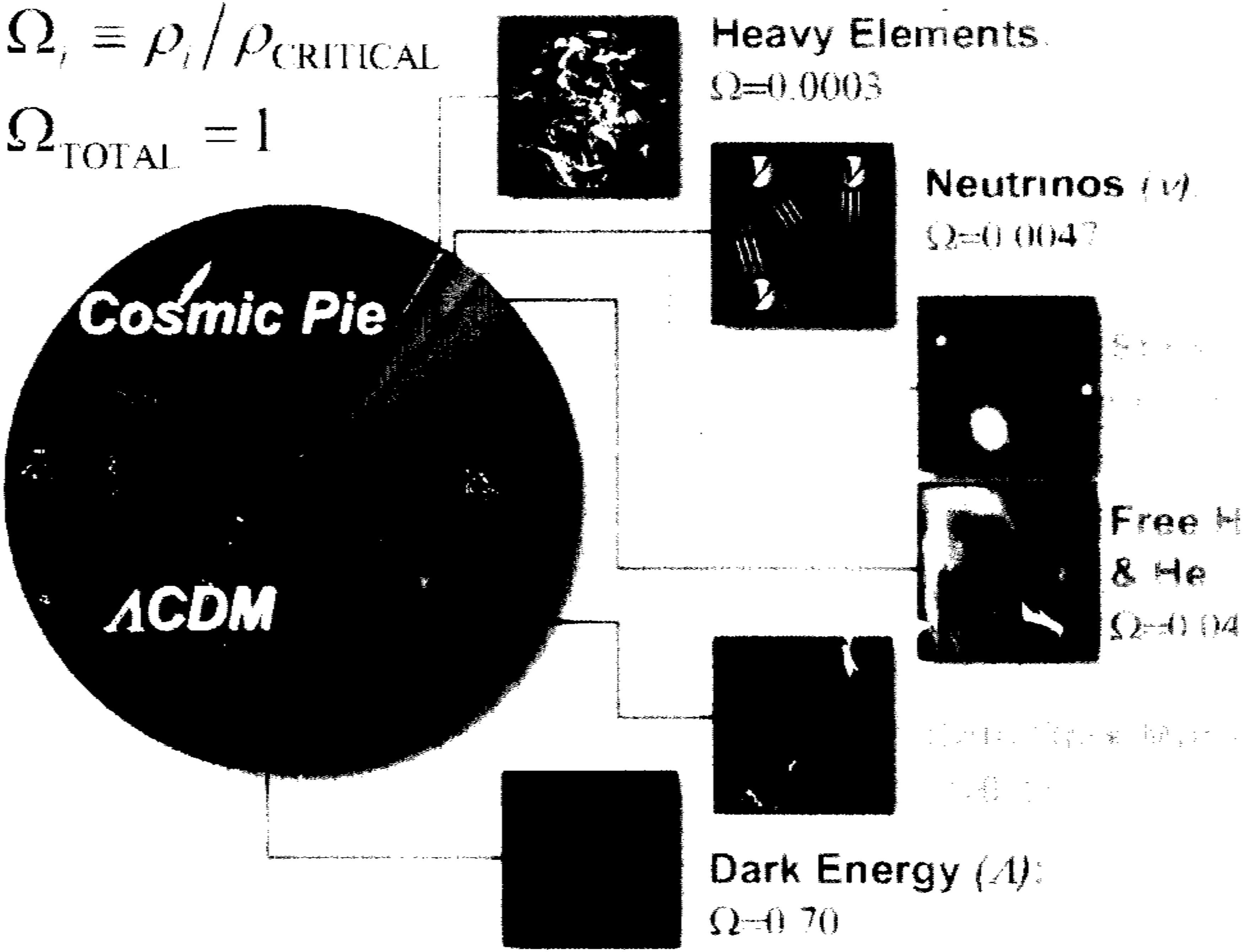
tightly constrained by Lorentz symmetry

$$\theta . \Lambda^2 \lesssim 10^{-17}$$

$\uparrow$
cutoff : $\Lambda = 1\,\text{TeV} \Rightarrow \theta < (10^{11} - 10^{12}\,\text{GeV})^{-2}$

$$\Omega_i \equiv \rho_i / \rho_{\mathrm{CRITICAL}}$$

$$\Omega_{\mathrm{TOTAL}} = 1$$



Cosmological "Constant"

a second number in general relativity?

a real constant?

$$\Lambda = \frac{\zeta(3)}{16\pi^2}\left(\frac{m_W}{m_P}\right)^8 \quad ????$$

to be calculated in M theory

approx. supersymmetry?

$$N=1 \Rightarrow \left(\frac{m_W}{m_P}\right)^2$$

more plausible than $\Lambda = 0$?

in obvious unbroken symmetries

relaxing towards zero?

$$\Lambda = f(t) \to 0, \quad \frac{p}{\rho} = w < 0$$

Universe not (yet) infinitely old, large

excitation of quantum gravity vacuum? (J.E. + Mavromatos + Nanopoulos

quintessence? (Steinhardt

relation to rest of physics??

an opportunity for theoretical physics!

(Little)

Too Much of a Good Thing!

observed: $\Lambda \lesssim 10^{-123} \cdot m_p^4$

QCD vacuum: $\Lambda \sim (100\ \text{MeV})^4$

$\sim 10^{-80}\ m_p^4$

Electroweak vacuum:

$\Lambda \sim (100\ \text{GeV})^4$

$\sim 10^{-68}\ m_p^4$

Supersymmetry breaking:

$\Lambda \sim (1\ \text{TeV})^4$

$\sim 10^{-64}\ m_p^4$

The biggest problem in physics?

much bigger than the hierarchy problem

need a theory of gravity